How to map an equation into computer code

I started with the equation for the total electric field for a horizontal electric dipole above an infinite, flat, perfect electric conductor, from Balanis Equation 4-116 :

$$E = j\eta \frac{k I_0 l}{4\pi r} e^{-jkr} j 2\sin(kh\cos\theta)\sqrt{1-\sin^2\theta\sin^2\phi} \qquad [1]$$

The first step is to clear out the imaginary parts. So I simplified things with the following substitutions.

$$A = 2\sin(kh\cos\theta) \qquad B = \sqrt{1-\sin^2\theta\sin^2\phi} \qquad Q = \eta\frac{k I_0 l}{4\pi r}$$

Which gives us:

$$E = jQe^{-jkr} jAB$$

Euler's identity: $e^{jx} = \cos x + j\sin x$

$$E = j^2 ABQ(\cos(-kr) + j\sin(-kr)) \qquad \text{apply Euler's identity}$$

The cosine function is symmetric around the origin: $\cos(-x) = \cos(x)$

$$E = j^2 ABQ(\cos(kr) + j\sin(-kr)) \qquad \text{cosine symmetry}$$

Squaring j gives us -1, while cubing it gives us -j for:

$$E = -1\,ABQ\cos(kr) - j\,ABQ\sin(-kr)$$

Eliminating the imaginary portion leaves us with:

$$E = -ABQ\cos(kr) \qquad \text{Go Lobos! [2]}$$

Back substitution gives us:

$$E = -\eta\frac{k I_0 l}{4\pi r}\cos(kr) 2\sin(kh\cos\theta)\sqrt{1-\sin^2\theta\sin^2\phi}$$

This represents the electric field potential at every point in volts per meter. Because I want to display the magnitude of the vector {E, θ, φ} I will ignore the minus sign. I will scale the magnitude for best presentation so the 2 is superfluous also. That leaves us with:

$$E = scale * \eta \frac{k I_0 l}{4 \pi r} \cos(kr) \sin(kh \cos\theta) \sqrt{1 - \sin^2\theta \sin^2\phi}$$ for the application.

My graphics system uses φ as elevation and θ as azimuth, the opposite of the text's convention. After I swap θ and φ the equation translates into the following computer code :

```
float E = scale * (ETA*k*I0*l) / (4.0*PI*r)                    after φ θ swap
        * cos(k*r) * sin(k*h * cos(RAD*phi))
        * sqrt(1 - sin(RAD*phi) * sin(RAD*phi) * sin(RAD*theta) * sin(RAD*theta));
```

where:

```
#define RAD 0.0174532925            // degree to radian conversion
#define PI      3.1415926535        // the ratio of circumference to diameter
#define ETA  376.991118308          // characteristic impedance of free space
float h = 1.3;                      // height of antenna in wavelengths
float I0 = 2.0;                     // maximum current in amperes
float r = 5000;                     // distance from antenna to observer in meters
float f = 100;                      // frequency in MHz
float lambda = 300 / f;             // wavelength in meters
float l = lambda / 2;               // length of antenna in meters
float k = TWOPI / lambda;           // save some space
float scale = 5000.0;               // scaling factor for display purposes
float dTheta = 3.0;                 // azimuth step size
float dPhi = 3.0;                   // elevation step size
```

[1] Balanis, Constantine A., Antenna Theory - Analysis and Design (John Wiley & Sons, Inc., 1997) page 176 equation 4-116
[2] University of New Mexico's mascot